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Hence,

$$4(a+r)^2(1-\cos\theta)^2 = (a^2+r^2-2ar\cos\theta)(3-2\cos\theta)^2,$$

or

$$(a^2+r^2)(4\cos\theta-5) + 2ar(4+\cos\theta-8\cos^2\theta+4\cos^3\theta) = 0,$$

or

$$(a-r)^2(4\cos\theta-5) = 2ar(1-\cos\theta)(1-2\cos\theta)^2.$$

Now the left side of this equation is essentially negative or zero; and the right side is essentially positive or zero. Hence the only real points on the locus are the points for which both sides are zero. These are the points $r = a, \theta = 0$; $r = a, \theta = 60^\circ$; $r = a, \theta = 300^\circ$.

Hence, the only triangles in the system are the two equilateral triangles on opposite sides of the given base.

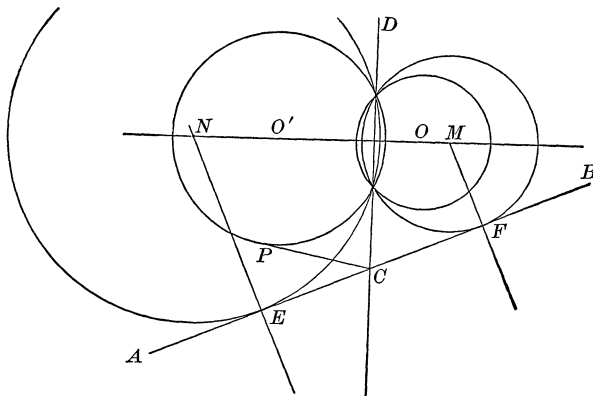
Note.—One other solution, unsigned, was received. This solution was based upon the relation between the radii of the in- and circumcircles and the distance between their centers, namely, $R^2 - d^2 = 2Rr$; since, in the given problem, $r = \frac{1}{2}R, d = 0$. Hence, the circles are concentric and the system of triangles reduces to two equilateral triangles on either side of the given line. EDITORS.

475. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Given two circles and a straight line, to draw a circle tangent to the line and coaxial with the two given circles.

SOLUTION BY GEO. W. HARTWELL, Hamline University.

Let O and O' be the two circles and AB the given straight line. Draw the radical axis CD of the two circles. From the point of intersection C of the axis and line AB , construct a tangent to O or O' . Then with C as a center and the length of this tangent, CP , as a radius lay off a distance CE and CF on AB . E and F will be points of tangency. The points, N and M , where perpendiculars erected to AB at E and F meet the line of centers OO' will be the centers of the required circles.



Since the required circle is coaxial with O and O' its center must be on the line of centers OO' . Because the distance along a tangent drawn from any point on the radical axis to each circle of the pencil is constant E or F must be the required point of tangency. Hence, there are two solutions; one circle with center M and radius MF , the other with center N and radius NE .

Also solved by ROGER JOHNSON and H. L. AGARD.

476. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Show that the locus of the middle points of a set of parallel chords intercepted between an hyperbola and its conjugate is $4b^2x^2y^2 - 4a^2y^4 = a^2b^4$.

SOLUTION BY J. A. CAPARO, University of Notre Dame.

Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (I) and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ (II) be the equations of the hyperbola and its conjugate respectively. Let m be the slope of the set of parallel chords and $P_1(x_1y_1)$, $P_2(x_2y_2)$ be the points where the chord $y = mx + c$ intersects the primary and conjugate hyperbolas respectively; and $P(xy)$ the middle point between P_1 and P_2 . Then

$$\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1 \quad (\text{I}), \quad \frac{x_2^2}{a^2} - \frac{y_2^2}{b^2} = -1 \quad (\text{II}),$$

$$y - y_1 = m(x - x_1) \quad (\text{III}), \quad x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2} \quad (\text{IV}).$$

From II and IV, we have:

$$\frac{(2x - x_1)^2}{a^2} - \frac{(2y - y_1)^2}{b^2} = -1,$$

which becomes

$$\frac{x^2 - xx_1}{a^2} - \frac{y^2 - yy_1}{b^2} = -\frac{1}{2}.$$

Combining this equation with III we get,

$$x - x_1 = \frac{a^2b^2}{2(a^2my - b^2x)};$$

from I and III we get,

$$\frac{x_1^2}{a^2} - \frac{[y - m(x - x_1)]^2}{b^2} = 1.$$

Eliminating x_1 between these two equations we have

$$\frac{1}{a^2} \left[x - \frac{a^2b^2}{2(a^2my - b^2x)} \right]^2 - \frac{1}{b^2} \left[y - \frac{a^2b^2m}{2(a^2my - b^2x)} \right]^2 = 1,$$

which, by I, reduces to

$$4 \left(\frac{x^2}{a^2} - \frac{y^2}{b^2} \right) \left(\frac{amy}{b} - \frac{bx}{a} \right)^2 = (a^2m^2 - b^2),$$

which is the general equation of the desired locus. As a particular case, if the chords are perpendicular to the x axis, then writing the equation in the form,

$$4 \left(\frac{x^2}{a^2} - \frac{y^2}{b^2} \right) \left(\frac{ay}{b} - \frac{bx}{am} \right)^2 = a^2 - \frac{b^2}{m^2}$$

and making $m = \infty$, we get

$$4b^2x^2y^2 - 4a^2y^4 = a^2b^4.$$

Also solved by PAUL CAPRON.

CALCULUS.

393. Proposed by LAENAS G. WELD, Pullman, Illinois.

Find the area of the least ellipse which can be drawn upon the face of a brick wall so as to inclose four bricks.

SOLUTION BY FRANK R. MORRIS, Glendale, Calif.

Let the ellipse be represented by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

then each quadrant will contain one brick. Also let m be the length and n the thickness of a brick. The ellipse must pass through the point (m, n) , i. e., the equation

$$\frac{m^2}{a^2} + \frac{n^2}{b^2} = 1 \text{ must be true.}$$

From this we get